Performance of Reduced-Complexity Transmit/Receive-Diversity Systems

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***Abstract*— We consider wireless systems with maximal-ratio combining diversity at the base station, and hybrid selection/maximal ratio combining at the mobile station. We have recently proposed these systems, which provide high-quality data links, while keeping the complexity at the mobile station low. In this paper, we analyze their performance in various realistic scenarios. Specifically, we analyze the influence of the number of base station antennas, of fading correlation and channel estimation errors. Furthermore, we explore the performance in channels measured in a microcellular environment. The simulation results confirm that the proposed scheme is effective in a variety of environments.**

***Index Terms*—Diversity, MIMO, antenna selection, channel estimation**

I. Introduction

Systems with multiple antennas at both transmitter and receiver have received considerable attention in recent years [1]. One approach to utilize multiple transmit antennas is to transmit different data streams from each antenna; these streams can be separated at the receiver side by the so-called BLAST schemes [2], [3]. However, this approach cannot be used with existing standards, as the requirement of backward-compatibility is not fulfilled. An alternative lies in the use of transmit and receive diversity purely for link-quality improvement, exploiting the diversity effect.[1] In such a system, the signals supplied to the different transmit antennas are weighted replicas of a single bit stream [5]; similarly, the receiver uses a linear combination of the signals obtained at the different receive antennas. The standard approach would be the use of maximal-ratio transmission (MRT) and maximal-ratio combining (MRC) at the transmitter and receiver respectively. It has been shown that with N_t transmit and N_r receive antennas, a diversity degree of $N_\mathrm{t}N_\mathrm{r}$ can be achieved [6]. Since it employs no special type of coding, any standard (single-antenna) receiver can detect the transmitted signal (albeit with a smaller diversity degree and thus reduced quality).

MRT (MRC) requires N_t (N_r) complete RF chains. There are numerous situations where this high degree of hardware expense is undesirable - this is especially important for the mobile station (MS). On the other hand, a simple (1 out of N) selection diversity gives considerably worse results. A compromise between these two possibilities is hybrid selection / maximum ratio combining (H-S/MRC[2] [7], [8], [9], [10]) where the best L out of N antennas are selected, and then combined, thus reducing the number of required RF chains to L.

In a recent paper [11], we have introduced this idea, and computed analytic performance bounds in i.i.d. Rayleigh fading channels. In this paper, we will analyze the system in more realistic environments (both modeled and measured), and investigate the influence of various system parameters on the performance. In Section II, we establish the channel and system model. Next, we investigate the influence of antenna correlation, number of antenna elements, and channel estimation errors on the performance. Section IV investigates the capacity in realistic (measured) channels. A summary wraps up the paper.

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[1] We assume in the following that the channel is known at the transmitter. This situation occurs either in a TDD system, or in a FDD system with a feedback of channel-state information. If the channel is unknown at the transmitter, space-time-codes [4] (which are not backward-compatible), delay diversity, or some similar scheme should be used.

[2] H-S/MRC in the following can denote either the transmission or the reception case.

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II. System and Channel Model

Figure 1 shows the generic system that we are considering - to simplify notation, we henceforth consider the uplink. A bit stream is sent through an encoder, and a modulator. A multiplexer switches the modulated signals to the best L_t out of N_t available antenna branches. For each selected branch, the signal is multiplied by a complex coefficient u whose actual value depends on the current channel realization. In a real system, the signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. For our model, we omit these stages, as well as their equivalents at the receiver, and treat the whole problem in equivalent baseband. Note, however, that it is exactly these parts that are most expensive and make the use of reduced-complexity systems desirable.

Next, the signal is sent over a quasi-static flat-fading channel. We also assume that the fading at all antenna elements is independent; its statistics obey a Rayleigh distribution. We denote the $N_r \times N_t$ matrix of the channel as

$$H = \begin{pmatrix} h_{11} & h_{12} & . \; . \; . & h_{1N_t} \\ h_{21} & h_{22} & . \; . \; . & h_{2N_t} \\ . & . & . \; . \; . & . \\ h_{N_r1} & h_{N_r2} & & h_{N_rN_t} \end{pmatrix}. \qquad (1)$$

If the channel is Rayleigh fading, the h_{ij} are independent identically distributed zero-mean, circularly symmetric complex Gaussian random variables with unit variance, i.e., the real and imaginary parts each have variance $1/2$. Consequently, the power carried by each transmission channel (h_{ij}) is chi-square distributed with 2 degrees of freedom.

The output of the channel is polluted by additive white Gaussian noise, which is assumed to be independent at all receiver antenna elements. The received signals are multiplied by complex weights w^* at all antenna elements (where superscript * denotes complex conjugation), and combined before passing to a decoder/detector. Details about the antenna selection procedure and the determination of the optimum weights can be found in [11].

III. Simulations

A. Simulation procedure

The computation of the capacity is not amenable to closed-form solutions. For the ideal (complex Gaussian i.i.d. case), we have presented upper and lower bounds in [11]. For the influence of nonidealities, we have to take

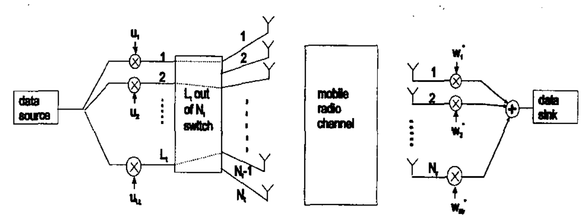

Fig. 1. System model

refuge to computer simulations. We first generate one realization of a MIMO channel transfer matrix. For the i.i.d, distributed case, this is trivial, as the entries are by definition just independent complex Gaussian random variables. Correlated entries can be created by multiplying the i.i.d. matrix with a matrix $\underline{A}$ that fulfills $\underline{A}\underline{A}^H = \underline{R}$, where $\underline{R}$ is the desired correlation matrix. We then create submatrices of size $L_t \times N_r$, by striking $(N_t - L_t)$ columns from the channel matrix. There are $\binom{N_t}{L_t}$ possible submatrices. For each submatrix, we compute the signal-to-noise ratio SNR (corresponding to the square of the largest singular value). Finally, we select the antenna combination (submatrix) that gives the largest SNR, and store it. This procedure is repeated N_{MC} times to give a statistical ensemble. The resulting SNR distribution forms the basis for all plots below.

Computation of the bit error probability (BEP) can be done by the classical method of averaging the "instantaneous BEP" (i.e., BEP for one given channel realization) over the statistics of the SNR. For coherent demodulation, this gives

$$P_e = K \int_0^\infty Q(\sqrt{a\gamma}) p_{\gamma_{tot}}(\gamma) d\gamma \qquad (2)$$

where the constants K and a depend on the modulation format [12].

For a capacity point of view, the whole system between encoder and decoder can be viewed as an effective scalar flat fading channel that is characterized by γ. The capacity from each channel realization is thus given by

$$C(\gamma) = \log_2(1 + \overline{\Gamma}\gamma). \qquad (3)$$

where $\overline{\Gamma}$ is the average SNR. Following the concept of "outage capacity" as discussed in [1], we can then define a statistical distribution of the capacity, assuming that for the transmission of each block of data (within the coherence time of the channel), we can have a different capacity value. Using standard techniques for functions of one ran-

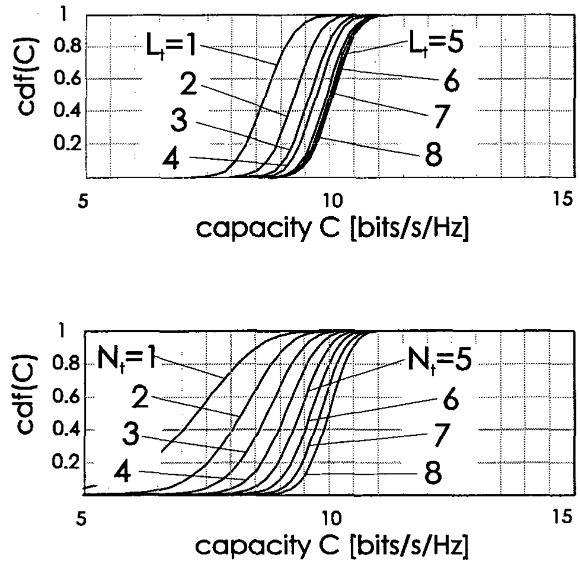

Fig. 2. Upper figure: Capacity of a system with H-S/MRT at the MS and MRC at the BS for various values of L_t with $N_t = 8$, $N_r = 2$, $SNR = 20$ dB. Lower figure: capacity of a system with MRT at BS and MRC at MS for varous values of N_t and $N_r = 2$, $SNR = 20$ dB.

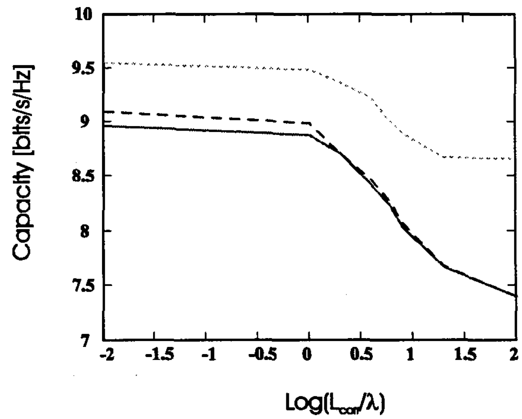

Fig. 3. 10% outage capacity of a system with 2 receiver antennas and H-S/MRT at the transmitter as a function of the antenna spacing. 3/8 system with optimum antenna selection (dashed), 3/8 system with antenna selection based on received power (solid) and 8/8 system (dotted).

dom variable [13], the pdf of the capacity becomes

$$p_C(C) = 2^C \frac{\ln(2)}{\overline{\Gamma}} p_{\gamma_{tot}} \left(\frac{2^C - 1}{\overline{\Gamma}} \right). \tag{4}$$

B. Simulation results

In this section, we present results from Monte Carlo simulations and discuss the influence of different system parameters on the performance. If not stated otherwise, we will use the following system parameters: $\overline{\Gamma} = 20$ dB, $N_r = 2$, $N_t = 8$, the modulation format is minimum shift keying (MSK), since it is commonly used in mobile radio systems.

Figure 2 shows the cumulative distribution of the capacity for different values of L_t. We see that the improvement by going from one to three antennas is larger than the gain going from three to eight. For comparison, we also show the capacity with pure MRT. The required number of RF chains is L_t for the HS-MRT case and N_t for the pure MRT case. Naturally, the capacity is the same for HS-MRT with $L_t = 8$, and MRT with $N_t = 8$. For a smaller number of RF chains, however, the hybrid scheme is much more effective, both in terms of diversity degree (slope of the curve) and mean capacity. This confirms the effectiveness of using HS-MRT.

Figure 3 shows the influence of correlation between the transmit antenna elements on the performance of the hybrid system. We show the 10% outage capacity of a $3/8$ system (i.e., $L_t = 3$, $N_t = 8$, with 2 receive antennas) with (i) optimum selection of the transmit antennas (i.e., choosing the transmit antennas that give the best SNR), (ii) with power-controlled selection of the transmit antennas (based on the choice of the transmit antenna that result in the highest receive power when all other transmit antennas are switched off; for details see [11]), and (iii) with MRT with $N_t = 8$. The outage capacity is plotted as a function of the ratio of correlation length of the channel to antenna spacing. We observe that the relative performance loss due to correlation is higher for the $3/8$ system than for the $8/8$ system. This can be explained by the fact that in a highly correlated channel, no diversity gain can be achieved, but all gain is due to beamforming. Thus antenna selection is ineffective, and the (beamforming) gain is only influenced by the number of actually used antenna elements. We furthermore observe that the difference between the SNR-based criterion for the antenna selection and the optimum antenna selection decreases as the correlation between the antennas increases, and vanishes at very large correlations. This makes sense, as the difference between the chosen antenna signals vanishes for highly correlated signals.

Figure 4 shows the influence of the number of antenna elements at the receiver. We find that as the number of receive antennas increases, the advantage of going from a 1/8 to a 8/8 system at the transmitter decreases. This is intuitively clear, as the beneficial effect of adding diversity antennas is smaller if there are already a lot of diversity

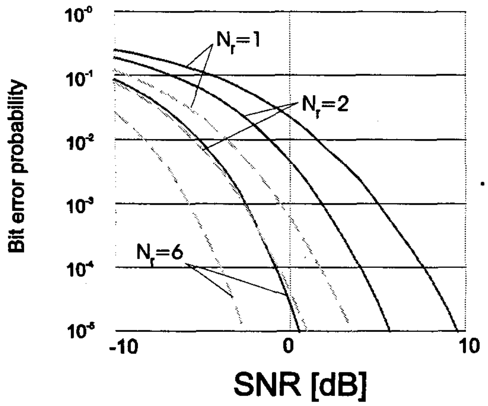

Fig. 4. Influence of the number of receive antennas on the BER. MSK modulation; $8/8$ (solid) and $1/8$ (dashed) H-S/MRT at transmitter.

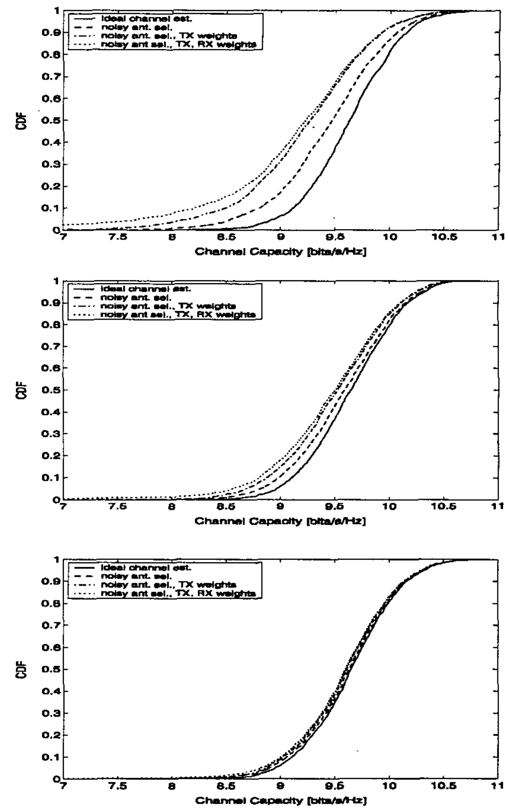

Fig. 5. Impact of errors in the estimation of transfer function matrix H. Cdf of the capacity for (i) ideal channel knowledge at TX and RX (solid), (ii) impaired antenna selection, but perfect antenna weights (dashed), (iii) impaired antenna weights at TX only (dotted), and (iv) impaired antenna weights at TX and RX (dash-dotted). Top plot: $SNR_{\text{pilot}} = 5$ dB, middle plot: $SNR_{\text{pilot}} = 10$ dB, bottom plot $SNR_{\text{pilot}} = 15$ dB.

antennas.

We have also investigated the influence of erroneous antenna selection on the capacity of the system. We assume that in a first stage, the complete channel transfer matrix is estimated. Based on that measurement, the antennas that are used for the actual data transmission are selected, and the antenna weights are determined. We distinguish four different cases: (i) perfect choice of the antennas and the antenna weights, (ii) impaired antenna selection, but perfect antenna weights (this can be achieved by measuring the transfer function of the actually selected antennas with a longer training sequence), (iii) imperfect choice of the antennas, as well as of the antenna weights at the transmitter, and perfect choice at the receiver (this is plausible if the feedback is done with finite precision and a finite lag), and (iv) imperfect choice of the antenna weights at transmitter and receiver. The errors in the transfer functions are assumed to have a complex Gaussian distribution with SNR_{pilot}. We found that measurement with an SNR of 10 dB results in a still tolerable loss of capacity (less than 5%). However, below that level, the capacity starts to decrease significantly. This is shown in Figure 5.

IV. Results in Measured Channels

We have also investigated the performance of our proposed scheme in measured channels. The measurements took place in a microcellular environment, specifically in a courtyard in Ilmenau, Germany. Four different measurement scenarios have been analyzed, and full details of the measurement scenarios can be found in [14]. For clarity only two scenarios are presented here, and they are[3]:

[3] Scenario I and II correspond to scenario II and III in [14], respectively.

Scenario I: Closed back-yard of size $34m \times 40m$ with inclined rectangular extension. The receiver array is situated in one rectangular corner with the array broad side pointing under 45^o inclination directly to the middle of the back-yard. The LOS connection between the transmitter and the receiver is $28m$.

Scenario II: Same back-yard as in scenario I, but with artificially obstructed LOS path. It is expected that the metallic objects generate serious multi-path and high order scattering that can only be observed within the dynamic range of the measurement system if the strong LOS path is obstructed.

In order to determine distributions of channel capacity and eigenvalues, a large number of measurements are required, which means a huge effort. Thus, for the measured channels we evaluate the different distributions by

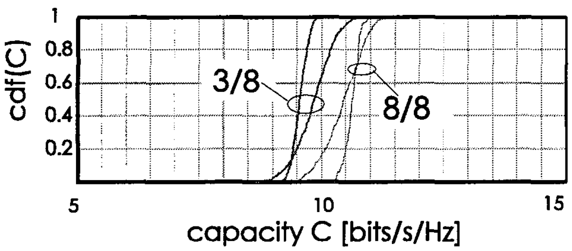

Fig. 6. Capacity of H-S/MRT system with $L_t = 3$ and $N_t = 8$ elements, and a MRT system with $N_t = 8$ elements in a microcellular environment. See text for description of the scenarios.

a method introduced in [15] , in order to keep the number of required measurements to a reasonable number. This method means that the double directional impulse response is measured, i.e. direction of departure, direction of arrival, time delay and power of the different taps. Then several impulse responses are *synthetically* generated from these measurements by assigning independent uniformly distributed $[0, 2\pi]$ random phases, α_k, to the different realizations of $h_{m,n}(f_q)$ as

$$h_{m,n}(f_q) = \sum_{k=1}^{K_q} A_k e^{j\phi_k} e^{-j\frac{2\pi}{\lambda}d(m\sin(\Omega_{R,k})+n\sin(\Omega_{T,k}))} e^{-j2\pi f_q \tau_k} e^{j\alpha_k} \tag{5}$$

where α_k stays unchanged as the different antenna elements are considered.

Figure 6 shows plots of the capacity for a 3/8 H-S/MRT system and an 8-element MRT scheme. The number of receive antennas is in both cases $N_r = 2$. We see that the performance that can be achieved in that environment is very close to the performance in i.i.d. channels, and sometimes it is even better.

V. Summary and Conclusions

We have investigated reduced-complexity wireless systems with transmit and receive diversity, using H-S/MRT at the MS, and MRC at the BS. Since the transceiver structure employs only weighted versions of the same signals, such a system is fully compatible with existing mobile radio systems, while the use of multiple antennas at both transmitter and receiver results in a high degree of diversity, using a limited number of receiver/transmitter RF chains.

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