Exact Symbol Error Probability for Optimum Combining in the Presence of Multiple Co-Channel Interferers and Thermal Noise

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Abstract—In this paper, we derive the exact symbol error probability for coherent detection of MPSK signals with optimum combining in the presence of multiple uncorrelated equal power co-channel interferers and thermal noise in a Rayleigh fading environment. The expression is general and valid for arbitrary numbers of receiving antennas or co-channel interferers. The complexity of the analytical model depends on the smaller of the number of antennas and the number of interferers.

I. Introduction

Adaptive arrays can significantly improve the performance of wireless communication systems by weighting and combining the received signals to reduce fading effects and suppress interference. In particular, with optimum combining the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise power ratio (SINR). This technique provides substantial improvement in performance over maximal ratio combining (where the received signals are combined to maximize the desired signal power only) when interference is present. However, determining the performance of optimum combining is more difficult than determining that of maximal ratio combining.

In this regard, closed-form expressions for the symbol error probability (SEP), have been derived for the single interferer case under the assumption of Rayleigh fading of the desired signal in [1] and with Rayleigh fading of the desired signal and interferer in [2].

With multiple interferers of arbitrary power, Monte Carlo simulation has been used to determine the SEP [1]. To avoid Monte Carlo simulation in case of equipower interferers, approximations have been presented in [3, 4]. However, the approximation of [3] still requires Monte Carlo simulation to derive mean eigenvalues (a table is provided in [3] for some cases), and the approximation of [4] has been proposed when the number of interferers is less then the number of antenna elements. The SEP expression is derived in [5], but the results are limited to the case of binary phase-shift keying (PSK) modulation, equal-power interferers and no thermal noise. In [6], upper bounds on the bit error probability of optimum combining were derived given the average power of the interferers. However, these bounds are generally not tight, typically 2 dB away from simulation results.

In this paper, starting from the eigenvalues distribution of Wishart complex matrices, we first give the exact expression of the SEP for coherent detection of M-ary PSK using optimum combining in the presence of multiple interferers as well as thermal noise in a flat Rayleigh fading environment.

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In Section II we provide the system description. A statistical analysis of the eigenvalues of the covariance matrix is given in Section III and the exact SEP is derived in Section IV. In Section V we show some numerical results and in Section VI we present some conclusions.

II. System Description

We consider coherent demodulation and optimum combining of multiple received signals in flat fading environment. The fading rate is assumed to be much slower than the symbol rate. Throughout the paper $(\cdot)^T$ is the transposition operator, and $(\cdot)^\dagger$ stands for conjugation and transposition. The received signal at N_{A}-element array output consists of desired signal, N_{I} interfering signals, and thermal noise. After matched filtering and sampling at the symbol rate, the array output vector at time k can be written as:

$$\mathbf{z}(k) = \sqrt{P_{\mathrm{D}}}\boldsymbol{c}_{\mathrm{D}} b_0(k) + \mathbf{z}_{\mathrm{IN}}(k)\,, \tag{1}$$

with

$$\mathbf{z}_{\mathrm{IN}}(k) = \sum_{j=1}^{N_{\mathrm{I}}} \sqrt{P_{\mathrm{I},j}}\boldsymbol{c}_{\mathrm{I},j} b_j(k) + \mathbf{n}(k)\,, \tag{2}$$

where P_{D} and $P_{\mathrm{I},j}$ are the mean (over fading) power of the desired signal and $j^{\underline{\mathrm{th}}}$ interferer, respectively; $\boldsymbol{c}_{\mathrm{D}} = [\boldsymbol{c}_{\mathrm{D},1}, \ldots, \boldsymbol{c}_{\mathrm{D},N_{\mathrm{A}}}]^T$ and $\boldsymbol{c}_{\mathrm{I},j} = [\boldsymbol{c}_{\mathrm{I},j,1}, \ldots, \boldsymbol{c}_{\mathrm{I},j,N_{\mathrm{A}}}]^T$ are the desired and $j^{\underline{\mathrm{th}}}$ interference propagation vectors, respectively; $b_0(k)$ and $b_j(k)$ (both with unit variance) are the desired and interfering data samples, respectively; and $\mathbf{n}(k)$ represents the additive noise. The symbols $b_0(k)$ are taken with equal probabilities. We model $\boldsymbol{c}_{\mathrm{D}}$ and $\boldsymbol{c}_{\mathrm{I},j}$ as multivariate complex-valued Gaussian vectors having $\mathbb{E}\{\boldsymbol{c}_{\mathrm{D}}\} = \mathbb{E}\{\boldsymbol{c}_{\mathrm{I},j}\} = \mathbf{0}$ and $\mathbb{E}\left\{\boldsymbol{c}_{\mathrm{D}}\boldsymbol{c}_{\mathrm{D}}^\dagger\right\} = \mathbb{E}\left\{\boldsymbol{c}_{\mathrm{I},j}\boldsymbol{c}_{\mathrm{I},j}^\dagger\right\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The additive noise is modeled as a white Gaussian random vector with independent and identically distributed (i.i.d.) elements with $\mathbb{E}\{\mathbf{n}(k)\} = \mathbf{0}$ and $\mathbb{E}\left\{\mathbf{n}(k)\mathbf{n}^\dagger(k)\right\} = \sigma^2\mathbf{I}$, where σ^2 is the thermal noise power per antenna element.

The SINR at the output of the N_{A}-element array with optimum combining can be expressed [1] as

$$\gamma = P_{\mathrm{D}}\boldsymbol{c}_{\mathrm{D}}^\dagger \mathbf{R}^{-1}\boldsymbol{c}_{\mathrm{D}}\,, \tag{3}$$

where the short-term covariance matrix $\mathbf{R}$, conditioned to all interference propagation vectors, is

$$\mathbf{R} = \mathbb{E}_{\mathbf{n},b_j(k)}\left\{\mathbf{z}_{\mathrm{IN}}(k) \cdot \mathbf{z}_{\mathrm{IN}}(k)^\dagger\right\} \tag{4}$$

and $\mathbb{E}_X\{\cdot\}$ denotes expectation with respect to X. The interference samples $b_j(k)$ (with $j = 1, \ldots, N_\text{I}$) are assumed to be Gaussian distributed [7]. Therefore,

$$\mathbf{R} = \sum_{j=1}^{N_\text{I}} P_{\text{I},j}\boldsymbol{c}_{\text{I},j}\boldsymbol{c}_{\text{I},j}^\dagger + \sigma^2\mathbf{I}. \tag{5}$$

It is important to remark that $\mathbf{R}$, and consequently also the SINR γ, varies at the fading rate.

The matrix $\mathbf{R}^{-1}$ can be written as $\mathbf{U}\Lambda^{-1}\mathbf{U}^\dagger$ where $\mathbf{U}$ is a unitary matrix and Λ is a diagonal matrix whose elements on the principal diagonal are the eigenvalues of $\mathbf{R}$, denoted by $(\lambda_1, \ldots, \lambda_{N_\text{A}})$. The vector $\mathbf{u} = \mathbf{U}^\dagger\boldsymbol{c}_\text{D} = [u_1, ..., u_{N_\text{A}}]^T$ has the same distribution as $\boldsymbol{c}_\text{D}$, since $\mathbf{U}$ represents a unitary transformation. The SINR given in (3) can be rewritten as:

$$\gamma = P_\text{D}\boldsymbol{c}_\text{D}^\dagger\mathbf{U}\Lambda^{-1}\mathbf{U}^\dagger\boldsymbol{c}_\text{D} = P_\text{D}\sum_{i=1}^{N_\text{A}}\frac{|u_i|^2}{\lambda_i}. \tag{6}$$

Since $\mathbf{R}$ is a random matrix, its eigenvalues are random variables too.

We now investigate the statistical properties of $(\lambda_1, \ldots, \lambda_{N_\text{A}})$. We will show later that this is related to problems arising in multivariate statistic, regarding the eigenvalues distribution of complex Wishart matrices. Let

$$\mathbf{C}_\text{I} \triangleq \begin{bmatrix} | & | & & | \\ \boldsymbol{c}_{\text{I},1} & \boldsymbol{c}_{\text{I},2} & \ldots & \boldsymbol{c}_{\text{I},N_\text{I}} \\ | & | & & | \end{bmatrix} \tag{7}$$

be a $(N_\text{A} \times N_\text{I})$ random matrix composed of N_I interference propagation vectors as columns. For equal power interferers, i.e., $P_{\text{I},j} = P_\text{I}$ for $j = 1, \ldots, N_\text{I}$, the expression (5) can be rewritten as

$$\mathbf{R} = P_\text{I}\tilde{\mathbf{R}} + \sigma^2\mathbf{I}, \tag{8}$$

where $\tilde{\mathbf{R}} = \mathbf{C}_\text{I}\mathbf{C}_\text{I}^\dagger$ is a $(N_\text{A} \times N_\text{A})$ random matrix. The eigenvalues of $\mathbf{R}$ can be written in terms of eigenvalues of $\tilde{\mathbf{R}}$, denoted by $(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_{N_\text{A}})$, as

$$\lambda_i = P_\text{I}\tilde{\lambda}_i + \sigma^2 \qquad i = 1, \ldots, N_\text{A}, \tag{9}$$

and therefore the knowledge of the joint p.d.f. of $(\lambda_1, \lambda_2, \ldots, \lambda_{N_\text{A}})$ can be derived by that of $(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_{N_\text{A}})$.

III. Derivation of the Joint Distribution of the Eigenvalues of $\mathbf{R}$

Let us consider the matrix $\tilde{\mathbf{R}}$, the joint probability density function (p.d.f.) of the N_A eigenvalues of $\tilde{\mathbf{R}}$ is given by the following theorem.

Theorem 1: The joint p.d.f. of the first $N_\text{min} \triangleq \min\{N_\text{A}, N_\text{I}\}$ *ordered* eigenvalues $\tilde{\boldsymbol{\lambda}} = [\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_{N_\text{min}}]^T$ of $\tilde{\mathbf{R}}$, with $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 \geq \ldots \geq \tilde{\lambda}_{N_\text{min}}$, is

$$f_{\tilde{\boldsymbol{\lambda}}}(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_{N_\text{min}}) = K\prod_{i=1}^{N_\text{min}} e^{-\tilde{\lambda}_i}\tilde{\lambda}_i^{N_\text{max}-N_\text{min}}.$$

$$\cdot\prod_{i=1}^{N_\text{min}-1}\left(\prod_{j=i+1}^{N_\text{min}}(\tilde{\lambda}_i - \tilde{\lambda}_j)^2\right), \tag{10}$$

where $N_\text{max} \triangleq \max\{N_\text{A}, N_\text{I}\}$ and K is the normalizing constant given by

$$K = \frac{\pi^{N_\text{min}(N_\text{min}-1)}}{\tilde{\Gamma}_{N_\text{min}}(N_\text{max})\tilde{\Gamma}_{N_\text{min}}(N_\text{min})}, \tag{11}$$

with

$$\tilde{\Gamma}_{N_\text{min}}(N_\text{max}) = \pi^{N_\text{min}(N_\text{min}-1)/2}\prod_{i=1}^{N_\text{min}}(N_\text{max} - i)!. \tag{12}$$

The additional $N_\text{A} - N_\text{min}$ eigenvalues of $\tilde{\mathbf{R}}$ are identically equal to zero.

Let us consider both the cases $N_\text{A} \leq N_\text{I}$ and $N_\text{A} > N_\text{I}$, separately.

Proof: of Theorem 1 [Case I. $N_\text{A} \leq N_\text{I}$]: When $N_\text{A} \leq N_\text{I}$, $\tilde{\mathbf{R}}$ can be related directly to the Wishart matrix [8,9], since the entries of the random matrix $\mathbf{C}_\text{I}$ form an i.i.d. Gaussian collection with zero-mean, independent real and imaginary parts, each with variance 1/2. So, we can write

$$\tilde{\mathbf{R}} = \sum_{j=1}^{N_\text{I}}\boldsymbol{c}_{\text{I},j}\boldsymbol{c}_{\text{I},j}^\dagger = \mathbf{C}_\text{I}\mathbf{C}_\text{I}^\dagger = \tilde{\mathbf{W}}(N_\text{A}, N_\text{I}), \tag{13}$$

where $\tilde{\mathbf{W}}(N_\text{A}, N_\text{I})$ is a $(N_\text{A} \times N_\text{A})$ complex Wishart matrix. Thus, the joint p.d.f. of the eigenvalues of $\tilde{\mathbf{R}}$ can be easily derived by [8, eq. (3.12) p. 37] and it is given by (10), (11) and (12). Note that the difference between (3.12) of [8] and (10), (11) depends on the fact that the Wishart Matrix considered in [8] is given by a collection of i.i.d Gaussian r.v.'s with independent real and imaginary parts having variance 1, whereas in our paper we assume a variance of 0.5. ■

Proof: of Theorem 1 [Case II. $N_\text{A} > N_\text{I}$]: Let us consider the following theorem.

Theorem 2: Suppose that $\mathbf{A} \in M_{m,p}$ and $\mathbf{B} \in M_{p,m}$ with $m \leq p$, the $(p \times p)$ matrix $\mathbf{BA}$ has the same m eigenvalues as the $(m \times m)$ matrix $\mathbf{AB}$, counting multiplicity, together with an additional $p - m$ eigenvalues identically equal to zero.

Proof: of Theorem 2 (See [10, p. 53]). ■

When $N_\text{A} > N_\text{I}$, $\tilde{\mathbf{R}}$ can still be related to the Wishart matrix, by means of Theorem 2. In fact, by introducing the $(N_\text{I} \times N_\text{A})$ matrix $\mathbf{A} \triangleq \mathbf{C}_\text{I}^\dagger$ and the $(N_\text{A} \times N_\text{I})$ matrix $\mathbf{B} \triangleq \mathbf{C}_\text{I}$, then the $(N_\text{A} \times N_\text{A})$ matrix $\mathbf{BA} = \mathbf{C}_\text{I}\mathbf{C}_\text{I}^\dagger = \tilde{\mathbf{R}}$ has the same N_I eigenvalues as the $(N_\text{I} \times N_\text{I})$ matrix $\mathbf{AB} = \mathbf{C}_\text{I}^\dagger\mathbf{C}_\text{I}$, and the additional $N_\text{A} - N_\text{I}$ eigenvalues are equal to zero. Moreover, since $N_\text{A} > N_\text{I}$, $\mathbf{C}_\text{I}^\dagger\mathbf{C}_\text{I} = \tilde{\mathbf{W}}(N_\text{I}, N_\text{A})$ is a $(N_\text{I} \times N_\text{I})$ complex Wishart matrix and therefore $\tilde{\mathbf{R}}$ has total of N_A eigenvalues, where N_I eigenvalues have the joint p.d.f. given by (10) with $N_\text{min} = N_\text{I}$ and $N_\text{max} = N_\text{A}$, and additional $N_\text{A} - N_\text{I}$ eigenvalues identically equal to zero. ■

Using the distribution theory for transformations of random vectors together with (9), the joint p.d.f. of $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_{N_\text{min}}]^T$ with $\lambda_1 \geq \ldots \geq \lambda_{N_\text{min}}$ is

$$f_{\boldsymbol{\lambda}}(\lambda_1, \ldots, \lambda_{N_\text{min}}) = \frac{1}{P_\text{I}^{N_\text{min}}}\cdot$$

$$\cdot f_{\tilde{\boldsymbol{\lambda}}}\left(\frac{\lambda_1-\sigma^2}{P_{\mathrm{I}}},\frac{\lambda_2-\sigma^2}{P_{\mathrm{I}}},\ldots,\frac{\lambda_{N_{\min}}-\sigma^2}{P_{\mathrm{I}}}\right), \tag{14}$$

where $f_{\tilde{\boldsymbol{\lambda}}}(\cdot)$ is given by Theorem 1. The additional $N_{\mathrm{A}}-N_{\min}$ eigenvalues of $\mathbf{R}$ are identically equal to σ^2.

IV. Derivation of the Symbol Error Probability

The SEP for optimum combining in the presence of multiple co-channel interferers and thermal noise in a fading environment is obtained by averaging the conditional SEP over the (desired and interfering signal) channel ensemble as

$$\begin{aligned} P_e &= \mathbb{E}_\gamma\{\Pr\{e \mid \gamma\}\} \\ &= \int_0^\infty \Pr\{e \mid \gamma = x\}\, f_\gamma(x)dx\,, \end{aligned} \tag{15}$$

where $\Pr\{e \mid \gamma\}$ is the SEP conditioned on the random variable γ, and $f_\gamma(\cdot)$ is the p.d.f. of the combiner output SINR. Although, the evaluation of (15) involves a single integration for averaging over the channel ensemble, it requires the knowledge of the p.d.f. of γ, which can be quite difficult to obtain. This is alleviated by using the chain rule of conditional expectation as

$$P_e = \mathbb{E}_{\boldsymbol{\lambda}}\left\{\underbrace{\mathbb{E}_{\mathbf{u}}\left\{\Pr\left\{e \mid \gamma = P_{\mathrm{D}}\sum_{i=1}^{N_{\mathrm{A}}}\frac{|u_i|^2}{\lambda_i}\right\}\right\}}_{P_{e|\boldsymbol{\lambda}}}\right\}, \tag{16}$$

where we first perform $\mathbb{E}_{\mathbf{u}}\{\cdot\}$ and average over the channel ensemble of the desired signal to obtain the conditional SEP, conditioned on the random vector $\boldsymbol{\lambda}$, denoted by $P_{e|\boldsymbol{\lambda}}$. We then perform $\mathbb{E}_{\boldsymbol{\lambda}}\{\cdot\}$ to average out the channel ensemble of the interfering signals.

The $j^{\underline{\text{th}}}$ interfering data samples, $b_j(k)$ $j=1,\ldots,N_{\mathrm{I}}$, can be modeled as a zero-mean, unitary variance Gaussian random variable. Note that the Gaussian assumption gives a good approximation when the interfering contribution is due to a large number of interferers sampled at random time but in any case it represents a worst case: here it will be used regardless of the number of interferers [7]. With this assumption together with the Gaussianity of $\mathbf{n}(k)$, $\Pr\{e \mid \gamma\}$ for coherent detection of M-ary PSK is given by [11]

$$\Pr\{e \mid \gamma\} = \frac{1}{\pi}\int_0^\Theta \exp\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}\gamma\right)d\theta\,, \tag{17}$$

where $c_{\mathrm{MPSK}} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. Using (17), $P_{e|\boldsymbol{\lambda}}$ can be written as

$$\begin{aligned} P_{e|\boldsymbol{\lambda}} &= \frac{1}{\pi}\int_0^\Theta \mathbb{E}_{\mathbf{u}}\left\{\exp\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}P_{\mathrm{D}}\sum_{i=1}^{N_{\mathrm{A}}}\frac{|u_i|^2}{\lambda_i}\right)\right\}d\theta \\ &= \frac{1}{\pi}\int_0^\Theta \psi_{\gamma|\boldsymbol{\lambda}}\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}\right)\,d\theta\,, \end{aligned} \tag{18}$$

where $\psi_{\gamma|\boldsymbol{\lambda}}(\cdot)$ is the characteristic function (c.f.) of γ, conditioned on $\boldsymbol{\lambda}$, given by

$$\psi_{\gamma|\boldsymbol{\lambda}}(j\nu) = \frac{1}{\prod_{i=1}^{N_{\mathrm{A}}}(1-j\nu P_{\mathrm{D}}/\lambda_i)}\,, \tag{19}$$

and we have used the fact that $\mathbf{u}$ is Gaussian with i.i.d. elements in deriving (19). Therefore, the conditional SEP, conditioned on $\boldsymbol{\lambda}$, in the general case of N_{A} antennas and N_{I} interferers becomes:

$$P_{e|\boldsymbol{\lambda}} = \frac{1}{\pi}\int_0^\Theta A(\theta)\prod_{i=1}^{N_{\min}}\left[\frac{\sin^2\theta}{\sin^2\theta + c_{\mathrm{MPSK}}\frac{P_{\mathrm{D}}}{\lambda_i}}\right]d\theta, \tag{20}$$

where

$$A(\theta) = \left[\frac{\sin^2\theta}{\sin^2\theta + c_{\mathrm{MPSK}}\frac{P_{\mathrm{D}}}{\sigma^2}}\right]^{N_{\mathrm{A}}-N_{\min}}. \tag{21}$$

Using (16) and (20), the unconditional SEP for optimum combining becomes

$$\begin{aligned} P_e = \mathbb{E}_{\boldsymbol{\lambda}}\{P_{e|\boldsymbol{\lambda}}\} = \int_0^\infty \cdots \int_{\lambda_3}^\infty \int_{\lambda_2}^\infty P_{e|\boldsymbol{\lambda}}\cdot \\ \cdot f_{\boldsymbol{\lambda}}(\boldsymbol{\lambda})\, d\lambda_1 d\lambda_2 \ldots d\lambda_{N_{\min}}\,. \end{aligned} \tag{22}$$

Expression (22) is exact and valid for arbitrary numbers of antennas and interferers; however, it requires the evaluation of nested $N_{\min}$-fold integrals, which can be cumbersome to evaluate for large $N_{\min}$. To give an idea of the amount of time needed for $N_{\min}$=2 (which allows to investigate either dual combining with any number of interferers or an arbitrary number of antennas and two interferers), the computation of (22) on a 450 MHz Personal Computer requires about 100 seconds. Considering that this is an exact result, the amount of time is quite acceptable.

V. Numerical Results

In this section, we investigate the effect of signal to noise ratio (SNR) defined as P_{D}/σ^2, the ratio between the desired received signal power and the total interfering power (SIR) defined as $P_{\mathrm{D}}/(N_{\mathrm{I}}\cdot P_{\mathrm{I}})$, the number of interferers, and the number of antenna branches on the SEP.

Fig. 1 shows the SEP as a function of SNR for dual optimum combining (N_{A}=2) with $M=2$, $N_{\mathrm{I}}=2$ and 4, and SIR = 0 dB. The results show excellent agreement between exact analysis and simulation results. The curves also exhibit an error floor when the number of interferers N_{I} is greater than the array degrees of freedom, i.e. $N_{\mathrm{A}}-1$, as expected. Fig. 2 shows the SEP as a function of SNR for different values of SIR ranging from -10 to 10 dB; M=4, N_{A}=4, N_{I}=2. Note that, since $N_{\mathrm{A}} > N_{\mathrm{I}}$, the antenna array is able to null out the interferers, and therefore there is no error floor in the performance. In Figs. 3 and 4 we have the SEP versus the SIR; N_{A}=2, M=4, SNR=5, 10 and 20 dB and several values of N_{I} are considered. Note that, when the interfering power is comparable with the thermal noise power, the number of interferers plays a marginal role (see Fig. 3 for $SIR > 10$ dB and Fig. 4 for $SIR > 5$ dB and 20 dB). Finally, the asymptotic SEP is limited by the thermal noise.

Fig. 5 shows the SEP as a function of the number of receiving antennas with SIR as a parameter ranging from -10 to 10 dB; M=4, N_{I}=2 and SNR=10 dB. The figure shows that the

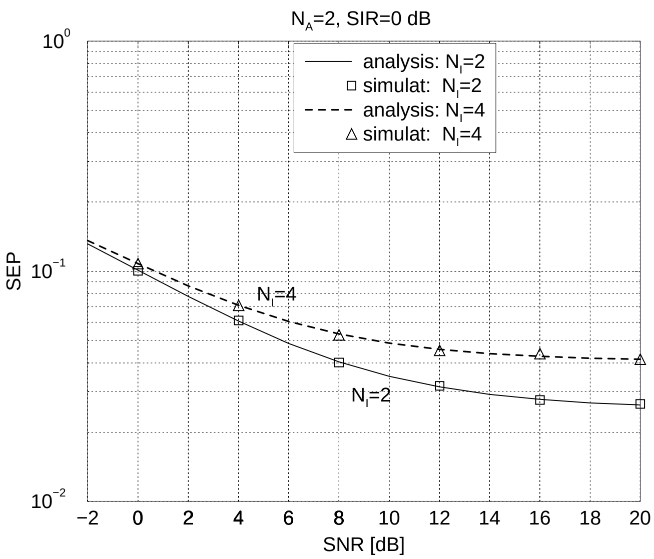

Fig. 1. The SEP as a function of SNR for N_{I}=2 and 4; M=2, SIR=0 dB. A comparison between analytical model and simulations

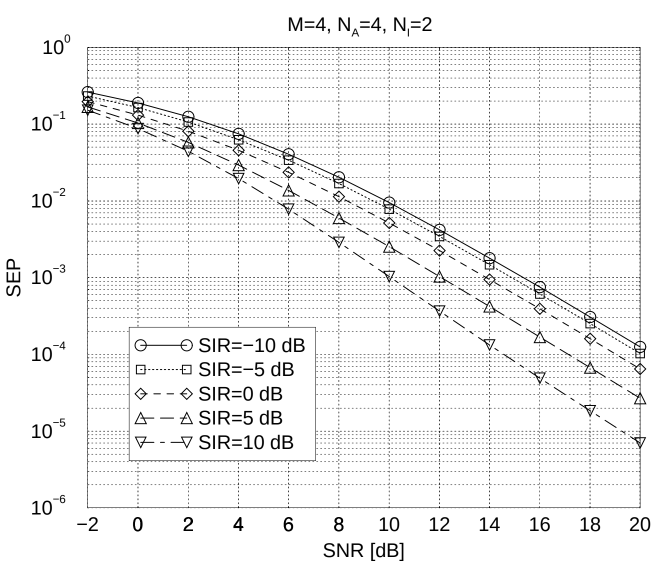

Fig. 2. The SEP as a function of SNR for different values of SIR; M=4, N_{A}=4, N_{I}=2.

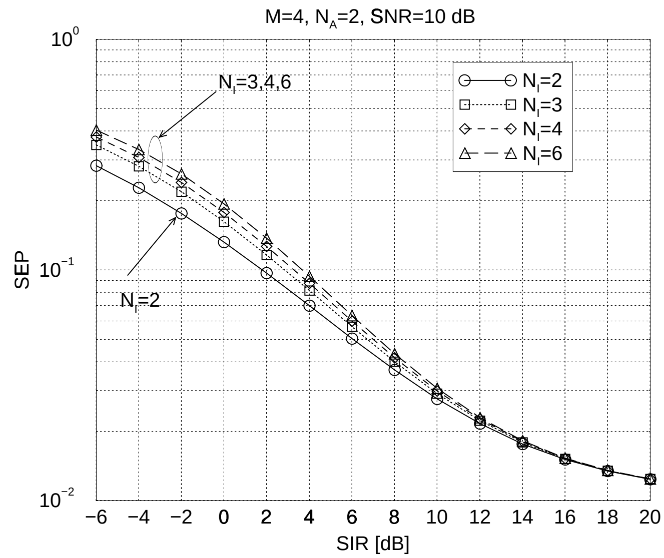

Fig. 3. The SEP as a function of SIR for N_{I} equal to 2, 3, 4 and 6; M=4, N_{A}=2, SNR=10 dB.

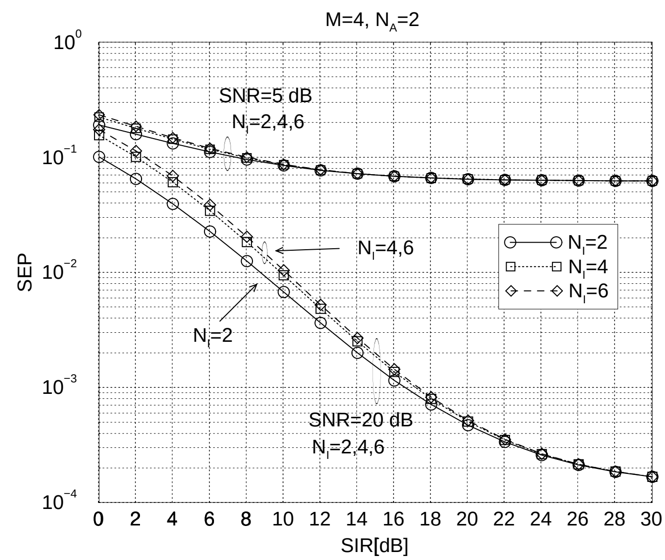

Fig. 4. The SEP as a function of SIR for N_{I} equal to 2, 4 and 6; M=4, N_{A}=2, SNR=5 and 20 dB.

system is able to exploit the spatial diversity provided by the increasing number of antennas (the SEP in logarithmic scale is approximately linear in N_{A}). Finally, in Fig. 6 we show the SEP versus the number of interferers N_{I}, for different values of SNR; dual optimum combining with SIR=5 dB and M=4 is considered. The figure confirms the results of Fig. 3; in fact, when the array is overloaded, the performance does not depend significantly on the number of interferers; this behavior is accentuated for small values of SNR.

VI. Conclusions

In this work, we have derived the exact symbol error probability for coherent detection of MPSK using optimum combining in the presence of multiple uncorrelated equal power interferers and thermal noise in a flat Rayleigh fading environment. Owing to the need to solve nested integrals, when the number of antennas and interferers become large, the analytical model tends to become cumbersome. On the other hand, in the case of dual combining or when the number of interferers is equal to two, the algorithm allows a fast evaluation of the SEP.

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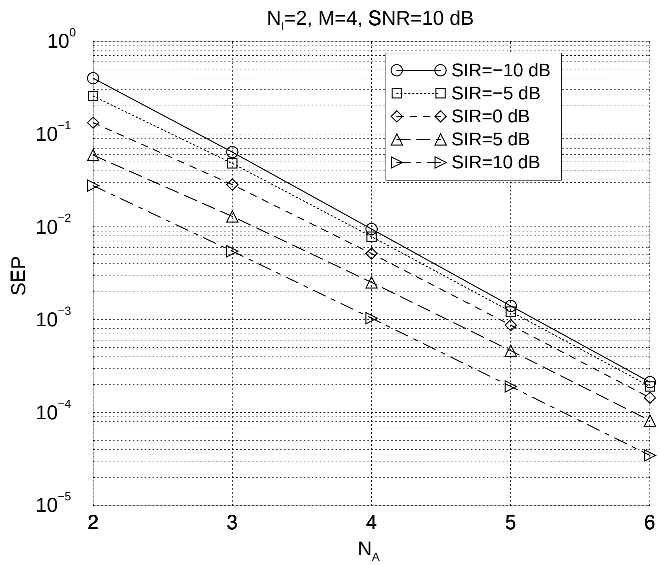

Fig. 5. The SEP as a function of N_{A} for different values of SIR; N_{I}=2, M=4, SNR=10 dB.

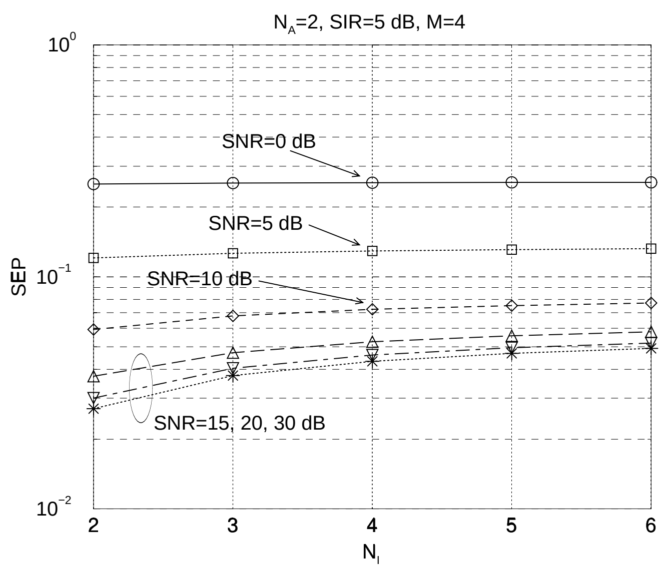

Fig. 6. The SEP as a function of N_{I} for different values of SNR; N_{A}=2, M=4, SIR=5 dB.

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