Exact Error Probability Expressions for MRC in Correlated Nakagami Channels with Unequal Fading Parameters and Branch Powers

Moe Z. Win and Jack H. Winters

Wireless Systems Research Department, AT&T Labs - Research
100 Schulz Drive, Red Bank, NJ 07701-7033 USA
Tel: (732)-345-3144 Fax: (732)-345-3038
e-mail: {win,jhw}@research.att.com

Abstract— **In this paper, we derive the symbol error probability (SEP) for maximal ratio combining with an arbitrary number of diversity branches in Nakagami fading with integer-order fading parameters, where the instantaneous signal-to-noise ratios (SNR's) of the diversity branches are *not* necessarily independent *or* identically distributed. We consider coherent detection of M-ary phase-shift keying and quadrature amplitude modulation. The proposed problem is made analytically tractable by: 1) transforming the physical diversity branches into a "virtual branch" domain; and 2) using alternative definite integral representations of the conditional SEP with finite limits; which results in a simple derivation. We further obtain a *canonical structure* for the SEP as a weighted sum of the elementary SEP's, which are the SEP's of a non-diversity (single-branch) system with appropriate fading parameters and average SNR, whose closed-form expressions are well-known.**

I. Introduction

With maximal ratio combining (MRC), the received signals from multiple diversity branches are cophased, weighted, and combined to maximize the output signal-to-noise ratio (SNR). Early work on the evaluation of symbol error probability (SEP) of MRC has mainly concentrated on Rayleigh and Rician channels [1], [2], [3]. Recently, Nakagami fading channels have received considerable attention in the study of the various aspects of wireless systems [4], [5], [6], [7], [8], [9], [10]. The Nakagami distribution, also known as the "m-distribution," provides greater flexibility in matching experimental data. Results have shown that the Nakagami distribution fits experimental data collected in a variety of fading environments better than Rayleigh, Rician, or log-normal distributions [11], [12], [13].

Analysis of MRC in Nakagami fading has typically assumed that diversity branches are independent [5], [7], [9]. Consideration of correlated fading has been limited only to dual-branch diversity [4], [5], with the exception of [8], [10]. The studies in [8], [10], though, consider only binary modulation with the assumption of equal m as well as average SNR's among all diversity branches. Furthermore, [8] assumed two specific correlation models, namely the equal-correlation and exponential-correlation models.

However, in some cases the average SNR is not necessarily equal for all the diversity branches and the fading statistics can also be different for each diversity branch. Some examples of unequal average SNR and/or fading statistics among all diversity branches are: 1) angle diversity using multiple beams where the average signal strength and fading statistics can be different in each beam; 2) polarization diversity with high base station antennas where for a vertically-polarized transmitter, the average received signal strength at the horizontally-polarized antenna is typically 6 to 10 dB lower than at the vertically-polarized antenna; 3) macrodiversity, where the shadow fading is different at each antenna and different local scattering conditions can lead to different fading statistics; and 4) Rake receivers, where the distribution of signal power with delay is not uniform and the first arriving multipath component is more likely to be specular than the later components. In these cases, exact expressions for the performance of MRC are not previously available in the literature.

In this paper, we derive the SEP for MRC with an arbitrary number of diversity branches in correlated Nakagami fading channels, where the instantaneous SNR's of the diversity branches are *not* necessarily independent *or* identically distributed. Specifically: 1) these SNR's can be arbitrarily correlated; 2) the SNR distributions can be from different Nakagami families, i.e., fading parameters (m's) are not necessarily equal; and 3) the average SNR's (averaged over the fading) of the branches are not necessarily equal. We consider coherent detection of M-ary phase-shift keying (MPSK) and quadrature amplitude modulation (MQAM). The proposed problem is made analytically tractable by: 1) transforming the physical diversity branches into a "virtual branch" domain; and 2) using alternative expressions for the conditional error probability. Note that we used the virtual branch technique in [14] to determine the mean and variance of the combiner output SNR for hybrid selection/maximal-ratio combining (H-S/MRC). We extended [14] to derive analytical expressions for the SEP with H-S/MRC for the case of identical branches in [15] and non-identical branches in [16].

The *canonical structure* of the SEP emerges from our derivation as a weighted sum of elementary SEP's, which are the SEP's of single-branch reception with appropriate

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fading parameters and average SNR, whose closed-form expressions are well-known. Thus, lengthy derivations are no longer needed for separate cases of wireless scenarios with different numbers of diversity branches and correlation models, as our results give a simple prescription for computing the parameters of single-branch Nakagami channels, the weights, and the number of terms in the sum. These results extend previously-derived results to cover numerous additional useful cases.

II. Diversity Combining Analysis

A. Preliminaries

Consider diversity reception in a correlated-fading environment. The output of the $i^{\underline{\text{th}}}$ branch is modeled as[1]

$$\tilde{r}_i(t) = \Re\left\{r_i(t)e^{j2\pi f_c t}\right\}\,, \tag{1}$$

where $r_i(t)$, $i = 1, \ldots, N$, is the equivalent lowpass (ELP) version of the $i^{\underline{\text{th}}}$ branch output, and f_c is the carrier frequency. The $i^{\underline{\text{th}}}$ branch ELP output is given by

$$r_i(t) = \alpha_i e^{-j\phi_i} s(t) + n_i(t)\,, \tag{2}$$

where $n_i(t)$ is an additive white Gaussian[2] noise (AWGN) process, assumed to be independent of the received signal, with two-sided power spectral density N_{0i}, $s_i(t)$ is the information-bearing signal with the average symbol energy E_s, α_i is the fading amplitude and ϕ_i is the phase of the $i^{\underline{\text{th}}}$ diversity branch. We model the α_i's as correlated Nakagami random variables (r.v.'s) with a *marginal* probability density function (p.d.f.) given by

$$f_{\alpha_i}(r) = \frac{2}{\Gamma(m_i)}\left(\frac{m_i}{\Omega_i}\right)^{m_i} r^{2m_i-1}e^{-m_i r^2/\Omega_i}, \tag{3}$$

where the fading parameter m_i denotes the Nakagami family, and $\Omega_i = \mathbb{E}\left\{\alpha_i^2\right\}$. Note that the α_i's can be from different Nakagami families, where m_i and Ω_i are not necessarily equal among the branches. We will refer to $(m_i, \Omega_i \frac{E_s}{N_{0i}})$ as Nakagami parameter pairs. As in [6], we assume that the m_i's are integers, noting that the measurement accuracy of the channel is typically only of integer order.

The instantaneous output SNR with MRC is given by [17]

$$\gamma_{\text{MRC}} = \sum_{i=1}^{N} \gamma_i\,, \tag{4}$$

where γ_i denotes the instantaneous SNR of the $i^{\underline{\text{th}}}$ diversity branch defined by $\gamma_i \triangleq \alpha_i^2 \frac{E_s}{N_{0i}}$. For a correlated Nakagami fading channel, the marginal p.d.f. of γ_i is given by

$$g_{\gamma_i}(x; m_i/\Gamma_i, m_i) = \frac{1}{\Gamma(m_i)}\left(\frac{m_i}{\Gamma_i}\right)^{m_i} x^{m_i-1}e^{-m_i x/\Gamma_i}, \tag{5}$$

where the SNR averaged over the fading in the $i^{\underline{\text{th}}}$ branch $\Gamma_i = \mathbb{E}\{\gamma_i\} = \mathbb{E}\left\{\alpha_i^2\right\}\frac{E_s}{N_{0i}} = \Omega_i \frac{E_s}{N_{0i}}$. Let $\gamma \triangleq (\gamma_1, \gamma_2, \ldots, \gamma_N)$ and denote the joint p.d.f. of $\gamma_1, \gamma_2, \ldots, \gamma_N$ by $f_\gamma(\{\gamma_i\}_{i=1}^N)$. In general,

$$f_\gamma(\{\gamma_i\}_{i=1}^N) \neq \prod_{i=1}^{N} g_{\gamma_i}(x; m_i/\Gamma_i, m_i) \tag{6}$$

since the γ_i's are correlated.

B. Virtual Branch Technique: The Key Idea

Conventional analysis of MRC in correlated Nakagami fading is, in general, cumbersome and complicated since: 1) the γ_i's can be correlated; 2) the γ_i's can be from different Nakagami families where the m_i's are not necessarily equal; and 3) the Γ_i's are not necessarily equal.[3] The difficulty described above is alleviated in the following by transforming the dependent physical branch variables into a new set of independent *virtual branches* and expressing the combiner output SNR as a linear function of the independent virtual branch SNR's.

Let X_i be the $2m_i \times 1$ vector defined by

$$X_i \triangleq [X_{i,1}\; X_{i,2}\; \cdots\; X_{i,2m_i}]^t\,, \qquad i = 1, 2, \ldots, N\,, \tag{7}$$

where $(\cdot)^t$ denotes transpose, and the elements $X_{i,k}$, are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance given by $\mathbb{E}\left\{X_{i,k}^2\right\} = \frac{\Gamma_i}{2m_i}$. Let X be the $D_T \times 1$ vector defined by

$$X \triangleq \left[X_1^t\; X_2^t\; \cdots\; X_N^t\right]^t\,, \tag{8}$$

where $D_T = \sum_{i=1}^{N} 2m_i$ is twice the sum of the Nakagami parameters.

By carefully constructing X, the statistical dependence among the N correlated branches can be related to the statistical dependence among the elements of X. When there is only second-order dependence, it suffices to construct the covariance matrix of X given by $K_X = \mathbb{E}\{XX^t\}$. Without loss of generality, one can assume that the γ_i's are indexed in increasing order of their Nakagami parameters, i.e., $m_1 \leq m_2 \leq \ldots \leq m_N$. We construct the correlation among the elements X such that

$$\mathbb{E}\{X_{i,k}X_{j,l}\} = \begin{cases} \frac{\Gamma_i}{2m_i} & \text{if } i = j \text{ and } k = l \\ \rho_{i,j}\sqrt{\frac{\Gamma_i}{2m_i}\frac{\Gamma_j}{2m_j}}\,, & \text{if } i \neq j \text{ but} \\ & k = l = 1, 2, \ldots, 2\min\{m_i, m_j\} \\ 0, & \text{otherwise}. \end{cases} \tag{9}$$

This construction implies that the $k^{\underline{\text{th}}}$ entries of X_i and X_j, with $i \neq j$, are correlated for $k = 1, 2, \ldots, 2\min\{m_i, m_j\}$. However, all the entries of X_i are mutually independent, and all other entries are independent. The relationship between the covariance of γ_i and γ_j and the covariance of

[1] The notation $\Re\{\cdot\}$ denotes the "real-portion" operator.

[2] The term "Gaussian" is used to denote "ELP complex circular Gaussian."

[3] Note that our model includes the case where only a *proper* subset of the branches have the same m_i's and/or Γ_i's. This is a subtle but important difference with previous studies where the analyses given in [5] and [8] required that the $\frac{m_i}{\Gamma_i}$'s are either *all* different or *all* equal.

the elements of X is given by:

$$\rho_{\gamma_i\gamma_j} \triangleq \frac{\mathbb{E}\{(\gamma_i - \mathbb{E}\{\gamma_i\})(\gamma_j - \mathbb{E}\{\gamma_j\})\}}{\sqrt{Var\{\gamma_i\}Var\{\gamma_j\}}} = \sqrt{\frac{\min\{m_i, m_j\}}{\max\{m_i, m_j\}}}\,\rho_{i,j}^2 . \quad (10)$$

The lower and upper bounds for the correlation between the two Nakagami branches are given by $0 \leq \rho_{\gamma_i\gamma_j} \leq \sqrt{\frac{\min\{m_i,m_j\}}{\max\{m_i,m_j\}}}$.[4]

Let $\{\lambda_l\}$ be the set of L *distinct* eigenvalues of K_X where each λ_l has algebraic multiplicity μ_l such that $\sum_{l=1}^{L}\mu_l = D_T = \sum_{i=1}^{N} 2m_i$. It can be shown that each γ_i is infinitely divisible [18], [19]. The infinite divisibility has implications on the *statistical* representation of the combiner output SNR as

$$\gamma_{\text{MRC}} \overset{L}{=} \sum_{l=1}^{L} \lambda_l V_l , \quad (11)$$

where the notation "$\overset{L}{=}$" denotes "equal in their respective distributions" (or "equal in their respective Laws") [18], [19], and the virtual branch variables V_l are *independent* chi-squared r.v.'s with μ_l degrees of freedom. In deriving (11), we have used the Karhunen-Loève (KL) expansion of the vector X [20].[5] The characteristic function of V_l is given by

$$\psi_{V_l}(j\nu) \triangleq \mathbb{E}\left\{e^{+j\nu V_l}\right\} = \left[\frac{1}{1-2j\nu}\right]^{\mu_l/2} . \quad (12)$$

Denoting $f_{V_l}(\cdot)$ as the p.d.f. of V_l and $V \triangleq (V_1, V_2, \ldots, V_L)$, the joint p.d.f. of $V_1, V_2, \ldots, V_L$ is,

$$f_V(\{v_l\}_{l=1}^{L}) = \prod_{l=1}^{L} f_{V_l}(v_l) . \quad (13)$$

III. Symbol Error Probability over the Channel Ensemble

The SEP for MRC in correlated Nakagami fading is obtained by averaging the conditional SEP over the channel ensemble. This can be accomplished by averaging the $\Pr\{e|\gamma_{\text{MRC}}\}$ over the p.d.f. of the γ_{MRC} as

$$P_e = \mathbb{E}_{\gamma_{\text{MRC}}}\{\Pr\{e|\gamma_{\text{MRC}}\}\} \quad (14)$$
$$= \int_0^{\infty} \Pr\{e|\gamma\}\, f_{\gamma_{\text{MRC}}}(\gamma)d\gamma , \quad (15)$$

[4]The fact that two Nakagami branches with *different* fading parameters m_i and m_j cannot be completely correlated (i.e., $\rho_{\gamma_i\gamma_j} < 1$) is not a drawback in our statistical representation, and it is just a manifestation of the basic fact that two r.v.'s with different distributions can not be completely correlated.

[5]A similar technique employing a frequency-domain KL expansion was used in [21] to study diversity combining in a frequency-selective Rayleigh-fading channel. Another technique similar to the KL expansion was also used in [22] to study the reception of noncoherent orthogonal signals in Rician and Rayleigh fading channels.

where $\Pr\{e|\gamma_{\text{MRC}}\}$ is the SEP conditioned on the random variable γ_{MRC}, and $f_{\gamma_{\text{MRC}}}(\cdot)$ is the p.d.f. of the combiner output SNR [23], [24]. Although, the evaluation of (15) involves a single integration for averaging over the channel ensemble, it requires the knowledge of the p.d.f. of γ_{MRC}. The required p.d.f. of the r.v γ_{MRC} to evaluate (15) is obtained in [8] for two specific correlation models, namely the equal correlation and exponential correlation models, with equal m as well as average SNR's among the diversity branches.

A. SEP for MPSK

For coherent detection of MPSK, an alternative representation $\Pr\{e|\gamma_{\text{MRC}}\}$, involving definite integral with *finite* limits, is given by [25], [26], [23], [27]

$$\Pr\{e_{\text{MPSK}}|\gamma_{\text{MRC}}\} = \frac{1}{\pi}\int_0^{\Theta} e^{-\frac{c_{\text{MPSK}}}{\sin^2\theta}\gamma_{\text{MRC}}} d\theta , \quad (16)$$

where $c_{\text{MPSK}} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. Evaluation of (14) can be accomplished, using the technique of [28], [29], by substituting the expression for γ_{MRC} directly in terms of the physical branch variables given by (4) into (16), and averaging over the physical branch variables. Therefore the SEP for MPSK becomes

$$P_{e,\text{MPSK}} = \frac{1}{\pi}\int_0^{\Theta} \mathbb{E}_{\{\gamma_i\}}\left\{e^{-\frac{c_{\text{MPSK}}}{\sin^2\theta}\sum_{i=1}^{N}\gamma_i}\right\} d\theta \quad (17)$$
$$= \frac{1}{\pi}\int_0^{\Theta}\int_0^{\infty}\cdots\int_0^{\infty} e^{-\frac{c_{\text{MPSK}}}{\sin^2\theta}\sum_{i=1}^{N}\gamma_i} \times f_{\gamma}(\{\gamma_i\}_{i=1}^{N})d\gamma_1\ldots d\gamma_N\, d\theta . \quad (18)$$

Note in (18) that, since the physical branches are *correlated*, direct use of the method given in [28], [29] requires an N-fold integration for the expectation operation in (17). This can be alleviated by expressing γ_{MRC} in terms of the virtual branch variables using (11) as:

$$P_{e,\text{MPSK}} = \frac{1}{\pi}\int_0^{\Theta} \mathbb{E}_{\{V_l\}}\left\{e^{-\frac{c_{\text{MPSK}}}{\sin^2\theta}\sum_{l=1}^{L}\lambda_l V_l}\right\} d\theta$$
$$= \frac{1}{\pi}\int_0^{\Theta}\int_0^{\infty}\cdots\int_0^{\infty} e^{-\frac{c_{\text{MPSK}}}{\sin^2\theta}\sum_{l=1}^{L}\lambda_l V_l} \times \prod_{l=1}^{L} f_{V_l}(v_l)dv_l\, d\theta . \quad (19)$$

Exploiting the fact that V_l's are independent, (19) becomes:

$$P_{e,\text{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{l=1}^{L}\mathbb{E}\left\{e^{-\frac{c_{\text{MPSK}}\lambda_l}{\sin^2\theta}V_l}\right\} d\theta$$
$$= \frac{1}{\pi}\int_0^{\Theta}\prod_{l=1}^{L}\psi_{V_l}\left(-\frac{c_{\text{MPSK}}\lambda_l}{\sin^2\theta}\right) d\theta . \quad (20)$$

The power of the virtual path technique is apparent by observing that the expectation operation in the above equation no longer requires N-fold integration.

Substituting (12) into (20) gives

$$P_{e,\text{MPSK}} = \frac{1}{\pi}\int_0^{\Theta}\prod_{l=1}^{L}\left[\frac{\sin^2\theta}{c_{\text{MPSK}}\,2\lambda_l + \sin^2\theta}\right]^{\mu_l/2} d\theta . \quad (21)$$

Thus the derivation of the SEP for the coherent reception of MPSK using N-branch MRC in correlated Nakagami fading reduces to a single integral over θ with finite limits, where the integrand is an L-fold product of simple expressions involving trigonometric functions with $L < N$.

B. SEP for MQAM

For coherent detection of MQAM with $M = 2^k$ for even k, $\Pr\{e|\gamma_{\text{MRC}}\}$ is given by [29]

$$\Pr\{e_{\text{MQAM}}|\gamma_{\text{MRC}}\} = q\,\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{c_{\text{MQAM}}}{\sin^2\theta}\gamma_{\text{MRC}}}d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4} e^{-\frac{c_{\text{MQAM}}}{\sin^2\theta}\gamma_{\text{MRC}}}d\theta, \quad (22)$$

where $q = 4(1-\frac{1}{\sqrt{M}})$, and $c_{\text{MQAM}} = \frac{3}{2(M-1)}$. Using the virtual branch technique, similar to the steps used for MPSK, the SEP for MQAM becomes

$$P_{e,\text{MQAM}} = q\,\frac{1}{\pi}\int_0^{\pi/2}\prod_{l=1}^{L}\left[\frac{\sin^2\theta}{c_{\text{MQAM}}\,2\lambda_l+\sin^2\theta}\right]^{\mu_l/2}d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\prod_{l=1}^{L}\left[\frac{\sin^2\theta}{c_{\text{MQAM}}\,2\lambda_l+\sin^2\theta}\right]^{\mu_l/2}d\theta. \quad (23)$$

Again, the derivation of the SEP for coherent reception of MQAM using MRC in correlated Nakagami fading reduces to two terms, each consisting of a single integral over θ involving a trigonometric function with finite limits.

IV. Canonical Expressions for SEP of MPSK and MQAM

The results given in (21) and (23) are the SEP for coherent detection of MPSK and MQAM, respectively, using N-branch MRC in correlated Nakagami channels. Specifically, the N diversity branches are correlated and m_i and Γ_i are not necessarily equal among the branches. The set $\{\lambda_l\}$ consists of L *distinct* eigenvalues of the covariance matrix K_X with each λ_l having algebraic multiplicity μ_l. The $\{\mu_l\}_{l=1}^{L}$ are related to the Nakagami parameters $\{m_i\}_{i=1}^{N}$ via $\sum_{l=1}^{L}\mu_l = \sum_{i=1}^{N}2m_i$.

Let us first consider the simplest possible case, namely single-branch reception (without diversity) in Nakagami fading with (m_1, Γ_1). For single-branch reception ($N = 1$), K_X is a $2m_1 \times 2m_1$ diagonal matrix having only one eigenvalue ($L = 1$) of multiplicity $\mu_1 = 2m_1$. The eigenvalue is $\lambda_1 = \mathbb{E}\{x_{1,j}^2\} = \frac{\Gamma_1}{2m_1}$. Substituting the appropriate parameters into (21), the SEP for single-branch reception of MPSK in Nakagami fading reduces to

$$P_{e,\text{MPSK}}(m_1,\Gamma_1) = \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\text{MPSK}}\frac{\Gamma_1}{m_1}+\sin^2\theta}\right]^{m_1}d\theta. \quad (24)$$

Note that (24) is equivalent to the SEP for reception of MPSK over Rayleigh-fading channels using MRC with m_1 *identical* branches having *equal* SNR's of $\frac{\Gamma_1}{m_1}$, and therefore a closed-form expression for (24) can be found in [30].

Similarly, the SEP for single-branch reception of MQAM can be reduced from (23) to

$$P_{e,\text{MQAM}}(m_1,\Gamma_1) = q\,\frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\sin^2\theta}{c_{\text{MQAM}}\frac{\Gamma_1}{m_1}+\sin^2\theta}\right]^{m_1}d\theta - \frac{q^2}{4}\frac{1}{\pi}\int_0^{\pi/4}\left[\frac{\sin^2\theta}{c_{\text{MQAM}}\frac{\Gamma_1}{m_1}+\sin^2\theta}\right]^{m_1}d\theta. \quad (25)$$

Note again that (25) is equivalent to the SEP for reception of MQAM over Rayleigh-fading channels using MRC with m_1 *identical* branches having *equal* SNR's of $\frac{\Gamma_1}{m_1}$, and therefore a closed-form expression for (25) can be found in [31].

The quest for obtaining insights from (21) and (23) is at its peak, which leads to an expansion of the integrand in (21). It can be shown that the integer μ_l is even for all l, and therefore (21) can be rewritten as

$$P_{e,\text{MPSK}} = \sum_{l=1}^{L}\sum_{k=1}^{\mu_l/2}A_{l,k}\,\frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{\text{MPSK}}\,2\lambda_l+\sin^2\theta}\right]^{k}d\theta, \quad (26)$$

where $A_{l,k}$'s are weighting coefficients of the expansion. Comparing (26) with (24)

$$P_{e,\text{MPSK}} = \sum_{l=1}^{L}\sum_{k=1}^{\mu_l/2}A_{l,k}\,P_{e,\text{MPSK}}(k, 2k\lambda_l). \quad (27)$$

Interesting insights can now be obtained from (27). The SEP for MPSK using N-branch MRC in correlated Nakagami fading, where m_i and Γ_i are not necessarily equal among the branches, is simply the weighted sum of the single-branch SEP's. The single-branch SEP for the (l,k)-entry is simply the SEP of single-branch MPSK reception over Nakagami channels with fading parameter k and $\Gamma_{l,k} = 2k\lambda_l$. Note also that the single-branch SEP for the (l,k)-entry is equivalent to the SEP of MPSK reception over Rayleigh-fading channels using MRC with k *identical* branches having *equal* SNR's of $2\lambda_l$.

Similarly, (23) can be rewritten as

$$P_{e,\text{MQAM}} = \sum_{l=1}^{L}\sum_{k=1}^{\mu_l/2}A_{l,k}\,P_{e,\text{MQAM}}(k, 2k\lambda_l). \quad (28)$$

Note that a similar structure, namely, a linear combination of the simple "elementary SEP's," is evident from (28) for MQAM. In other words the SEP for MQAM reception with N correlated branches is simply the sum of the weighted SEP's for single-branch MQAM reception over Nakagami channels with parameter pairs $(k, 2k\lambda_l)$.

V. Conclusions

We derived the symbol error probability for coherent detection of MPSK and MQAM using maximal ratio combining with an arbitrary number of diversity branches in Nakagami fading with integer-order fading parameters, where the instantaneous SNR's of the diversity branches are *not* necessarily independent *or* identically distributed.

The proposed problem was made analytically tractable by: 1) transforming the physical diversity branches into the "virtual branch" domain; and 2) using alternative definite integral representations of the conditional SEP's for MPSK and MQAM with finite limits; which resulted in a simple derivation for the SEP in correlated Nakagami fading. We further obtained a *canonical structure* for the SEP as a weighted sum of the elementary SEP's, which are the SEP's of the non-diversity (single-branch) system with appropriate fading parameters and average SNR.

Thus, lengthy derivations are no longer needed for each wireless scenario with different numbers of diversity branches and correlation models, as our results give a simple prescription for computing the parameters of single-branch Nakagami channels, the weights, and the number of terms used in the sum to calculate the result. Our results extend previously-derived results to cover numerous additional useful cases.

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